Haar null convex sets

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A Borel set E in an abelian Polish group X is Haar null if there is a Borel probability measure μ on X such that $\mu(x + E) = 0$ for every $x \in X$. Haar null sets are a generalisation of sets with zero Haar measure to nonlocally compact, abelian Polish groups and, as in the locally compact case, form a translation-invariant σ-ideal. We present several geometric, measure-free characterisations of Haar null closed, convex sets in separable Banach spaces, generalising the well-known fact that a closed, convex subsets of $\mathbb{R}^d$ has zero Lebesgue measure if and only if it has empty interior.