On the numerical index of the real two-dimensional L_p space

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The concept of numerical index was introduced by Lumer in 1968 in the context of the study and classification of operator algebras. This is a constant relating the norm and the numerical range of bounded linear operators on the space. Namely, the numerical index of a Banach space X, $n(X)$, is the greatest constant $k \geq 0$ such that

$$k\|T\| \leq \sup\{|x^*(Tx)| \colon x \in X,\ x^* \in X^*,\ \|x\| = \|x^*\| = x^*(x) = 1\}$$

for every operator $T \in \mathcal{L}(X)$.

There are some classical Banach spaces for which the numerical index has been calculated, however the problem of computing the numerical index of L_p-spaces when $p \neq 1, 2, \infty$ has been latent since the beginning of the theory. The aim of this talk is to give an overview of the state of this problem, paying special attention to the two-dimensional real case for which we have recently been able to give a partial solution. More precisely, we have calculated the numerical index of the two-dimensional real L_p-space for $6/5 \leq p \leq 6$. ([1], [2]).

Acknowledgements Research partially supported by projects PGC2018-093794-B-I00 (MCIU/AEI/FEDER, UE), P20-00255 (Junta de Andalucía/FEDER, UE), A-FQM-484-UGR18 (Junta de Andalucía/FEDER, UE), and FQM-185 (Junta de Andalucía/FEDER, UE); and by the Ph.D. scholarship FPU18/03057 (MECD).

References

[1] J. Merí and A. Quero, On the numerical index of absolute symmetric norms on the plane, *Linear Mult. Algebra* **69** (2021), no. 5, 971–979. DOI: 10.1080/03081087.2020.1762532.

[2] J. Merí and A. Quero, On the numerical index of the real two-dimensional L_p space, *Linear Mult. Algebra.* Published online (2023). DOI: 10.1080/03081087.2023.2181938.