Non linear geometry and asymptotic properties of Banach spaces

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The non linear geometry of Banach spaces aims at finding linear properties of Banach spaces that are stable under non linear embeddings or equivalences and at characterizing them in purely metric terms. This is what the celebrated Ribe program is about for local properties of Banach spaces (i.e., isomorphic properties of their finite dimensional subspaces). Linear asymptotic properties of Banach spaces, on the other hand, are those that can be read on their finite codimensional subspaces, like the properties of the weakly null sequences or nets, or more generally of weakly null trees. In the last twenty five years, many asymptotic properties have been proved to be stable under nonlinear embeddings or equivalences. In order to minimize the overlap with a recent doc-course given in Granada on this subject, we will briefly summarize the relevant linear asymptotic properties and mainly address the study of universal Banach spaces for separable metric spaces and large scale embeddings such as coarse and coarse-Lipschitz embeddings. In one direction, it will involve the use of special Lipschitz free spaces. In the other direction, we will describe the use of Kalton's interlacing graphs and its recent developments. If time allows, we will also describe old and new applications of the Gorelik principle to coarse equivalences.