From property (beta) of Rolewicz to metric geometry

Denka Kutzarova

University of Illinois at Urbana-Champaign

In connection to well-posedness of optimization problems, in 1987 Stefan Rolewicz introduced a new geometric property of the norm of a Banach space and called it (β). The spaces which admit an equivalent norm with property (β) formed a new isomorphic class between super-reflexive and reflexive spaces. This area of research got a new life in a paper of Baudier, Kalton and Lancien from 2010 where they gave a metric characterization of asymptotic super-reflexive spaces, which turned out to be exactly the new isomorphic class of spaces with an equivalent norm with property (β). In a paper from 2012, Lima and Andrianarivony showed the importance of property (β) for solving a ten-year old question of Bates, Johnson, Lindenstrauss, Preiss and Schechtman about uniform (nonlinear) quotients. Independently, also in 2012, Revalski and Zhivkov defined the notion of compact uniform convexity in connection to the study of metric projections, and that turned out to be (isometrically) the same as the property (β) of Rolewicz. That led to the introduction of asymptotic midpoint uniform convexity (AMUC), and later it was proved that there are no uniform bi-Lipschitz embeddings of some types of countably branching graphs(for example, countably branching diamond and Laakso graphs) into Banach spaces with an equivalent AMUC norm. The lecture will be a short survey of these results, including several papers of the author with different sets of coauthors, both from the first and the second period of this research.