Planes in Schatten-p, and a problem of Ball, Carlen and Lieb

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In his seminal 1956 paper Olof Hanner proved an inequality, now known as Hanner's inequality, that captures the modulus of uniform convexity of L_p. Similar inequality was later considered by Ball, Carlen and Lieb in 1994 for non-commutative variants of L_p, Schatten-p, and while they managed to sidestep this defect, Ball, Carlen and Lieb only managed to prove Hanner's inequality for Schatten-p when $p \geq 4$ or $1 \leq p \leq 4/3$.

We propose a new conjecture on Schatten-p spaces: every two-dimensional subspace of Schatten-p is linearly isometric to a subspace of L_p. This would immediately imply Hanner's inequality for Schatten-p, together with wide range of other inequalities. We prove this conjecture when $p = 3$ using some curious matrix identities.