Riemann Integration and Asymptotic Structure of Banach Spaces

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Let X be a Banach space. If every Riemann-integrable function $f \colon [0,1] \to X$ is Lebesgue almost everywhere continuous, then X is said to have the Lebesgue property. A longstanding open problem in the geometry of Banach spaces is to derive a condition that is both necessary and sufficient in order for X to have the Lebesgue property. In this talk, I will give a brief overview of the history of work done on this problem and past results, and I will then present its recent solution (due to B. Sari and myself, and independently to M. Pizzotti). It turns out that the Lebesgue property is equivalent to an asymptotic structure in X that is strictly between the classical notions of spreading and asymptotic models. I will also discuss several other results (some due to B. Sari and myself) that help to place the Lebesgue property in its proper context with respect to asymptotic structures.