Generic Banach spaces

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Let X be a separable (finite or infinite-dimensional) Banach space. Consider a game where two players alternately play finite-dimensional subspaces of X so that they produce an infinite sequence $(X_n)_{n\in\mathbb{N}}$, where X_n $(1+\varepsilon_n)$-isometrically embeds into X_{n+1} and (ε_n) go sufficiently fast to 0 so that the direct limit Y of the sequence $(X_n)_{n\in\mathbb{N}}$ exists. In some cases the second player has a game strategy to guarantee that the limit Y is isometric to X. We provide several characterizations of such Banach spaces X, e.g. based on the action of the linear isometry group of X, based on the topological complexity of the isometry class of X in the space of separable Banach spaces, etc. Examples of such spaces include $L_p([0,1])$, for $p \in [1,\infty)$, the Gurarii space, and $L_p([0,1], L_q)$ for some $p \neq q$. This is joint work with Marek Cúth and Noé de Rancourt.