

Metric Embeddings of Laakso Graphs Into Banach Spaces

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Let X be a Banach space which is not super-reflexive. Then, for each $n \geq 1$ and $\varepsilon > 0$, we exhibit metric embeddings of the Laakso graph \mathcal{L}_n into X with distortion less than $2 + \varepsilon$ and into $L_1[0, 1]$ with distortion $4/3$. These results improve previous estimates although we do not know whether they are optimal. However, we show that the distortion of an embedding of \mathcal{L}_2 (respectively, the diamond graph D_2) into $L_1[0, 1]$ is at least $9/8$ (respectively, $5/4$).

We also present some results and open questions on the the Banach-Mazur distance to ℓ_1^N of the transportation cost spaces of diamond and Laakso graphs.