Symplectic structures on Rochberg spaces

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A real Banach space X is said to be *symplectic* if there is a continuous alternating bilinear map $\omega : X \times X \to \mathbb{R}$ such that the induced map $L_\omega : X \to X^*$ given by $L_\omega(x)(y) = \omega(x, y)$ is an isomorphism onto. N. Kalton and R. Swanson showed that the celebrated Kalton-Peck space Z_2 is a symplectic space with no Lagrangian subspaces. In this talk we will consider the sequence of higher order Rochberg spaces $\mathfrak{R}^{(n)}$ obtained from the interpolation scale of ℓ_p spaces, which can be considered as generalizations of both ℓ_2 and Z_2 since $\mathfrak{R}^{(1)} = \ell_2$ and $\mathfrak{R}^{(2)} = Z_2$. We will show that all these spaces $\mathfrak{R}^{(n)}$, $n > 1$, are symplectic and contain no (infinite dimensional) Lagrangian subspaces; in other words, they admit a nontrivial symplectic structure. The talk is based on joint work with J. M. F. Castillo, M. González and R. Pino.

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