On the numerical index of 2-dimensional Lipschitz-free spaces

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The numerical index of a given Banach space is a constant relating the geometry of the norm and the numerical radius of bounded linear operators on the space.

This concept appeared for the first time in the literature after its introduction by G. Lumer in 1968, and has been widely studied since then. Nowadays, although the numerical index of certain classes of Banach spaces—like $L_1(\mu)$ or $C(K)$—are well known due to their concrete properties, finding the numerical index of a given Banach space is still a challenging task, even for 2-dimensional spaces.

Along this talk, we provide the explicit formula for the numerical index of any 2-dimensional Lipschitz-free space $\mathcal{F}(M)$, also giving the construction of operators attaining this value as its numerical radius. As a consequence, the numerical index of 2-dimensional Lipschitz-free spaces can take any value of the interval $[\frac{1}{2}, 1]$, and this whole range of numerical indices can be attained by taking 2-dimensional subspaces of any Lipschitz-free space of the form $\mathcal{F}(A)$, where $A \subset \mathbb{R}^n$ with $n \geq 2$ is any set with non-empty interior.

This is based on a joint work with Antonio José Guirao and Vicente Montesinos—[1].

References

[1] Ch. Cobollo, A.J. Guirao, and V. Montesinos, *The numerical index of 2-dimensional Lipschitz-free spaces*, Available at `https://arxiv.org/abs/2304.13183` (2023).

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