Zero product determined Banach algebras and Jordan homomorphisms

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A (complex) Banach algebra A is said to be *zero product determined* if every continuos bilinear map $\varphi\colon A \to \mathbb{C}$ with the property that $\varphi(a,b) = 0$ whenever $ab = 0$ is of the form $\varphi(a,b) = \tau(ab)$ for some continuous linear map $\tau \in A^*$.

A rather wide class of Banach algebras that includes C^*-algebras and group algebras, among others, are zero product determined, and we use this property to characterize Jordan homomorphisms on these algebras. More precisely, we show that an operator T from a Banach algebra A onto a Banach algebra with BAI is a Jordan homomorphism in any of the following cases:

- A is a C^*-algebra or the algebra of approximable operators on a Banach space which dual has the bounded approximation property and T satisfies $T(a)T(b) + T(b)T(a) = 0$ if $ab = ba = 0$.
- A is a group algebra associated with a compact group and T satisfies $T(a)T(b) = T(b)T(a) = 0$ if $ab = ba = 0$.