XXII Escuela Lluís Santaló

Linear and Non-Linear Analysis in Banach Spaces

July 17-21
Santander
Palacio de la Magdalena

MINI-COURSES

Verónica Dimant
Gilles Lancien
Abraham Rueda Zoca
Richard Smith

SPEAKERS

R. Aliaga
A. Avilés
Ch. Cobollo
W. Cuellar
S. Dilworth
M. Doucha
V. Ferenczi
H. Gaebler
D. Kutzarova
E. Kopecká
J. Langemets
G. Martínez Cervantes
R. Medina
J. Orihuela
M. Ostrovskii
C. Petitjean
A. Quero de la Rosa
A. Quilis
J. Rondoš
T. Russo
D. Sobota
J. Somaglia
T. Veeorg

Real Sociedad Matemática Española

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Organizing committee:
Sheldon Gil Dantas (Universitat de València)
Luis C. García Lirola (Universidad de Zaragoza)

About the Lluís Santaló School

The **XXII Lluís Santaló School: Linear and Non-linear Analysis in Banach Spaces** is especially (but not exclusively!) aimed at young researchers and PhD students in Banach Space Theory and related areas. It will include four mini-courses as well as talks by junior and senior researchers.

The School is organized and sponsored by the Royal Spanish Mathematical Society (**RSME**) and Menéndez Pelayo International University (**UIMP**) and also sponsored by **Fundación R. Areces**.

Location

The School will be held in the historic building of the **Palacio de la Magdalena** which was in the past a summer residence for the Spanish Royal Family and currently used to host summer courses through the Menéndez Pelayo International University.

School dinner will be held in La Casa de Revert (Av. del Stadium) on Wednesday evening.

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Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:00 - 10:00	Opening	V. Dimant	A. Rueda Zoca	G. Lancien	G. Lancien
10:00 - 11:00	V. Dimant	G. Lancien	R. Smith	A. Rueda Zoca	J. Orihuela
11:00 - 12:00	Break R. Smith	Break	Break	Break	Break G. Martínez-Cervantes
12:00 - 13:00	A. Rueda Zoca	R. Smith	M. Doucha C. Petitjean	V. Dimant	A. Avilés
13:00 - 14:00	Lunch	E. Kopecká Lunch	Lunch	V. Ferenczi Lunch	M. Ostrovskii Lunch and closing
14:00 - 15:00					
15:00 - 16:00	T. Russo	J. Langemets	Poster session D. Kutzarova	S. Dilworth	
16:00 - 17:00	R. Aliaga	R. Medina A. Quero	J. Somaglia T. Veeorg	J. Rondoš D. Sobota	
17:00 - 18:00	W. Cuellar	Ch. Cobollo	A. Quilis	H. Gaebler	

Monday 17th

9.30 - 10.00	Opening
10.00 - 10.50	Verónica Dimant *Linearization of non-linear functions I*
11.00 - 11.30	Break
11.30 - 12.20	Richard Smith *Lipschitz-free spaces and representing measures I*
12.30 - 13.20	Abraham Rueda Zoca *Geometry of tensor products and bilinear mappings in Banach spaces I*
13.30 - 15.30	Lunch
15.30 - 16.10	Tommaso Russo *Norming tales*
16.20 - 17.00	Ramón J. Aliaga *The Schur and Radon-Nikodým properties for Lipschitz-free spaces*
17.05 - 17.25	Wilson Cuellar *Symplectic structures on Rochberg spaces*

Tuesday 18th

9.30 - 10.20	Verónica Dimant *Linearization of non-linear functions II*
10.30 - 11.20	Gilles Lancien *Non linear geometry and asymptotic properties of Banach spaces I*
11.30 - 12.00	Break
12.00 - 12.50	Richard Smith *Lipschitz-free spaces and representing measures II*
13.00 - 13.30	Eva Kopecká *Convergence of remote projections onto convex sets*
13.35 - 15.30	Lunch
15.30 - 16.10	Johann Langemets *On slicely countably determined sets in Banach spaces*
16.20 - 16.40	Rubén Medina *A nonlinear Enflo example*
16.45 - 17.05	Alicia Quero *On the numerical index of the real two-dimensional L_p space*
17.10 - 17.30	Christian Cobollo *On the numerical index of 2-dimensional Lipschitz-free spaces*

Wednesday 19th

9.30 - 10.20	Abraham Rueda Zoca *Geometry of tensor products and bilinear mappings in Banach spaces II*
10.30 - 11.20	Richard Smith *Lipschitz-free spaces and representing measures III*
11.30 - 12.00	Break
12.00 - 12.40	Michal Doucha *Generic Banach spaces*
12.50 - 13.30	Colin Petitjean *On the weak topology in Lipschitz free spaces*
13.35 - 15.00	Lunch
15.00 - 15.30	Poster session
15.30 - 16.00	Denka Kutzarova *From property (beta) of Rolewicz to metric geometry*
16.10 - 16.30	Jacopo Somaglia *Some recent progresses in renorming theory*
16.35 - 16.55	Triinu Veeorg *Unconditional bases and Daugavet renormings*
17.00 - 17.20	Andrés Quilis *Different convexity concepts in norms of Banach spaces*

Thursday 20th

9.30 - 10.20	Gilles Lancien *Non linear geometry and asymptotic properties of Banach spaces II*
10.30 - 11.20	Abraham Rueda Zoca *Geometry of tensor products and bilinear mappings in Banach spaces III*
11.30 - 12.00	Break
12.00 - 12.50	Verónica Dimant *Linearization of non-linear functions III*
13.00 - 13.30	Valentin Ferenczi *On isometry groups on the space of converging sequences*
13.35 - 15.30	Lunch
15.30 - 16.00	Stephen Dilworth *Metric embeddings of Laakso graphs into Banach spaces*
16.10 - 16.30	Jakup Rondoš *Distances between $C(K)$ spaces*
16.35 - 16.55	Damian Sobota *Continuous operators from spaces of Lipschitz functions*
17.00 - 17.20	Harrison Gaebler *Riemann Integration and Asymptotic Structure of Banach Spaces*

Friday 21st

9.30 - 10.20	Gilles Lancien *Non linear geometry and asymptotic properties of Banach spaces III*
10.30 - 11.00	José Orihuela *Separable Faces and Renormings of non-separable Banach Spaces*
11.10 - 11.40	Break
11.40 - 12.10	Gonzalo Martínez Cervantes *On the complemented subspace problem*
12.20 - 12.50	Antonio Avilés *Banach spaces and Banach lattices*
13.00 - 13.30	Mikhaill Ostrovskii *Dvoretzky-type theorem for locally finite subsets of a Hilbert space*
13.35 - 15.30	Lunch and closing

Abstracts

Minicourses

Linearization of non-linear functions

Verónica Dimant
Universidad de San Andrés

In Banach space theory the most prominent class of functions are the continuous linear mappings since they preserve the structure of the underlying spaces. When dealing with a space of non-linear functions, it is convenient to apply a procedure that associates it with a space of linear mappings (defined on a more complex domain). In this course we present some characteristics of a general procedure and then focus on the linearization of spaces of bilinear mappings, homogeneous polynomials and bounded holomorphic functions.

Non linear geometry and asymptotic properties of Banach spaces

Gilles Lancien
Université de Franche-Comté

The non linear geometry of Banach spaces aims at finding linear properties of Banach spaces that are stable under non linear embeddings or equivalences and at characterizing them in purely metric terms. This is what the celebrated Ribe program is about for local properties of Banach spaces (i.e., isomorphic properties of their finite dimensional subspaces). Linear asymptotic properties of Banach spaces, on the other hand, are those that can be read on their finite codimensional subspaces, like the properties of the weakly null sequences or nets, or more generally of weakly null trees. In the last twenty five years, many asymptotic properties have been proved to be stable under nonlinear embeddings or equivalences. In order to minimize the overlap with a recent doc-course given in Granada on this subject, we will briefly summarize the relevant linear asymptotic properties and mainly address the study of universal Banach spaces for separable metric spaces and large scale embeddings such as coarse and coarse-Lipschitz embeddings. In one direction, it will involve the use of special Lipschitz free spaces. In the other direction, we will describe the use of Kalton's interlacing graphs and its recent developments. If time allows, we will also describe old and new applications of the Gorelik principle to coarse equivalences.

Geometry of tensor products and bilinear mappings in Banach spaces

Abraham Rueda Zoca
Universidad de Granada

The aim of this course is to introduce the projective tensor product from the natural linearisation property of bilinear and continuous mappings. By the geometric spirit of this course and the difficulty of making explicit computations with the projective norm, we will pay a special attention to the identification of the dual of the projective tensor product as a space of bilinear and continuous mappings.

After this is done, we will focus on analysing a selection of questions on projective tensor products and on the space of bilinear and continuous mappings of geometric nature.

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Lipschitz-free spaces and representing measures

Richard Smith
University College Dublin

Let M be a complete metric space with base point 0, and let $\mathrm{Lip}_0(M)$ denote the Banach space of Lipschitz functions on M that vanish at 0. The *Lipschitz-free* space $\mathcal{F}(M) \subseteq \mathrm{Lip}_0(M)^*$ over M is defined as the closed linear span of the set of functionals $f \mapsto f(x)$, $x \in M$, and is an isometric predual of $\mathrm{Lip}_0(M)$. These spaces have important applications in the linear and nonlinear theory of Banach spaces.

Let $\widetilde{M} = \{(x, y) \in M \times M : x \neq y\}$ and denote by $\beta\widetilde{M}$ its Stone-Čech compactification. Define the *de Leeuw transform* $\Phi : \mathrm{Lip}_0(M) \to C(\beta\widetilde{M})$ by

$$\Phi f(x, y) = \frac{f(x) - f(y)}{d(x, y)}, \quad f \in \mathrm{Lip}_0(M),\ (x, y) \in \widetilde{M},$$

and extending continuously to $\beta\widetilde{M}$. This is a linear isometric embedding whose dual $\Phi^* : C(\beta\widetilde{M})^* \to \mathrm{Lip}_0(M)^*$ is therefore a quotient map. Thus, every element of $\mathrm{Lip}_0(M)^*$ can be represented (non-uniquely) by Radon measures on $\beta\widetilde{M}$ via the de Leeuw transform.

Given $\phi \in \mathrm{Lip}_0(M)^*$, there exists a positive Radon measure μ on $\beta\widetilde{M}$ such that $\Phi^*\mu = \phi$ and $\|\mu\| = \|\phi\|$. We call such a measure μ an *optimal representation* of ϕ. In this mini-course, we investigate the properties of optimal representations μ such that $\mu(\widetilde{M}) = \|\mu\|$ (in which case $\Phi^*\mu \in \mathcal{F}(M)$) or more generally $\mu(\widetilde{M}) > 0$. We illustrate connections with cyclical monotonicity from optimal transport theory, elements of $\mathcal{F}(M)$ that can be induced by measures on M, and present an application of this work to the extreme point problem in Lipschitz-free spaces.

This is joint work with Ramón Aliaga (Universitat Politècnica de València) and Eva Pernecká (Czech Technical University, Prague).

Talks

The Schur and Radon-Nikodým properties for Lipschitz-free spaces

Monday, 16.20-17.00

Ramón J. Aliaga
Universidad Politécnica de Valencia

The problem of determining which Lipschitz-free spaces $\mathcal{F}(M)$ over complete metric spaces M have the Radon-Nikodým and Schur properties has been around since Kalton's seminal paper from 2004. In a joint work with Chris Gartland, Colin Petitjean and Tony Procházka, we recently solved the problem and proved that both properties are actually equivalent for the class of Lipschitz-free spaces. They hold precisely when M is purely 1-unrectifiable, that is, when it contains no bi-Lipschitz copy of a subset of the real line with positive measure. The keys to this result lie in the analysis of locally flat Lipschitz functions, and in the reduction of the problem to the case of compact metric spaces M, where these properties turn out to also be equivalent to $\mathcal{F}(M)$ being a dual Banach space.

Banach spaces and Banach lattices

Friday, 12.20-12.50

Antonio Avilés
Universidad de Murcia

In this talk we will overview some recent developments in the theory of Banach lattices.

On the numerical index of 2-dimensional Lipschitz-free spaces

Tuesday, 17.10-17.30

Christian Cobollo
Universidad Politécnica de Valencia

The numerical index of a given Banach space is a constant relating the geometry of the norm and the numerical radius of bounded linear operators on the space.

This concept appeared for the first time in the literature after its introduction by G. Lumer in 1968, and has been widely studied since then. Nowadays, although the numerical index of certain classes of Banach spaces—like $L_1(\mu)$ or $C(K)$—are well known due to their concrete properties, finding the numerical index of a given Banach space is still a challenging task, even for 2-dimensional spaces.

Along this talk, we provide the explicit formula for the numerical index of any 2-dimensional Lipschitz-free space $\mathcal{F}(M)$, also giving the construction of operators attaining this value as its numerical radius. As a consequence, the numerical index of 2-dimensional Lipschitz-free spaces can take any value of the interval $[\frac{1}{2}, 1]$, and this whole range of numerical indices can be attained by taking 2-dimensional subspaces of any Lipschitz-free space of the form $\mathcal{F}(A)$, where $A \subset \mathbb{R}^n$ with $n \geq 2$ is any set with non-empty interior.

This is based on a joint work with Antonio José Guirao and Vicente Montesinos—[1].

References

[1] Ch. Cobollo, A.J. Guirao, and V. Montesinos, *The numerical index of 2-dimensional Lipschitz-free spaces*, Available at `https://arxiv.org/abs/2304.13183` (2023).

Symplectic structures on Rochberg spaces

Monday, 17.05-17.25

Wilson Cuellar
Universidade de São Paulo

A real Banach space X is said to be *symplectic* if there is a continuous alternating bilinear map $\omega : X \times X \to \mathbb{R}$ such that the induced map $L_\omega : X \to X^*$ given by $L_\omega(x)(y) = \omega(x, y)$ is an isomorphism onto. N. Kalton and R. Swanson showed that the celebrated Kalton-Peck space Z_2 is a symplectic space with no Lagrangian subspaces. In this talk we will consider the sequence of higher order Rochberg spaces $\mathfrak{R}^{(n)}$ obtained from the interpolation scale of ℓ_p spaces, which can be considered as generalizations of both ℓ_2 and Z_2 since $\mathfrak{R}^{(1)} = \ell_2$ and $\mathfrak{R}^{(2)} = Z_2$. We will show that all these spaces $\mathfrak{R}^{(n)}$, $n > 1$, are symplectic and contain no (infinite dimensional) Lagrangian subspaces; in other words, they admit a nontrivial symplectic structure. The talk is based on joint work with J. M. F. Castillo, M. González and R. Pino.

This work was partially supported by Fapesp grants (2016/25574-8) and (2019/23669-0).

Metric Embeddings of Laakso Graphs Into Banach Spaces

Thursday, 15.30-16.00

Stephen Dilworth
University of South Carolina

Let X be a Banach space which is not super-reflexive. Then, for each $n \geq 1$ and $\varepsilon > 0$, we exhibit metric embeddings of the Laakso graph $\mathcal{L}_n$ into X with distortion less than $2 + \varepsilon$ and into $L_1[0, 1]$ with distortion $4/3$. These results improve previous estimates although we do not know whether they are optimal. However, we show that the distortion of an embedding of $\mathcal{L}_2$ (respectively, the diamond graph D_2) into $L_1[0, 1]$ is at least $9/8$ (respectively, $5/4$).

We also present some results and open questions on the the Banach-Mazur distance to ℓ_1^N of the transportation cost spaces of diamond and Laakso graphs.

Generic Banach spaces

Wednesday, 12.00-12.40

Michal Doucha
Czech Academy of Sciences

Let X be a separable (finite or infinite-dimensional) Banach space. Consider a game where two players alternately play finite-dimensional subspaces of X so that they produce an infinite sequence $(X_n)_{n\in\mathbb{N}}$, where X_n $(1 + \varepsilon_n)$-isometrically embeds into X_{n+1} and (ε_n) go sufficiently fast to 0 so that the direct limit Y of the sequence $(X_n)_{n\in\mathbb{N}}$ exists. In some cases the second player has a game strategy to guarantee that the limit Y is isometric to X. We provide several characterizations of such Banach spaces X, e.g. based on the action of the linear isometry group of X, based on the topological complexity of the isometry class of X in the space of separable Banach spaces, etc. Examples of such spaces include $L_p([0, 1])$, for $p \in [1, \infty)$, the Gurarii space, and $L_p([0, 1], L_q)$ for some $p \neq q$. This is joint work with Marek Cúth and Noé de Rancourt.

On isometry groups on the space of converging sequences

Thursday, 13.00-13.30

Valentin Ferenczi
Universidade de São Paulo

The space c_0 is a hyperplane of, and therefore is complemented into, the space c of convergent sequences. We shall explain that in some sense related to compatibility with isometry groups, the space c_0 is badly complemented into c. Time allowing, we shall relate this to a theory of bounded actions of groups on exact sequences of Banach spaces, and explain how the example of c_0 is a counterexample to a result on exact sequences of reflexive spaces. Based on works of Antunes-Ferenczi-Grivaux-Rosendal and of Castillo-Ferenczi.

Riemann Integration and Asymptotic Structure of Banach Spaces

Thursday, 17.00-17.20

Harrison Gaebler
University of North Texas

Let X be a Banach space. If every Riemann-integrable function $f\colon [0,1] \to X$ is Lebesgue almost everywhere continuous, then X is said to have the Lebesgue property. A longstanding open problem in the geometry of Banach spaces is to derive a condition that is both necessary and sufficient in order for X to have the Lebesgue property. In this talk, I will give a brief overview of the history of work done on this problem and past results, and I will then present its recent solution (due to B. Sari and myself, and independently to M. Pizzotti). It turns out that the Lebesgue property is equivalent to an asymptotic structure in X that is strictly between the classical notions of spreading and asymptotic models. I will also discuss several other results (some due to B. Sari and myself) that help to place the Lebesgue property in its proper context with respect to asymptotic structures.

From property (beta) of Rolewicz to metric geometry

Wednesday , 15.30-16.00

Denka Kutzarova
University of Illinois at Urbana-Champaign

In connection to well-posedness of optimization problems, in 1987 Stefan Rolewicz introduced a new geometric property of the norm of a Banach space and called it (β). The spaces which admit an equivalent norm with property (β) formed a new isomorphic class between super-reflexive and reflexive spaces. This area of research got a new life in a paper of Baudier, Kalton and Lancien from 2010 where they gave a metric characterization of asymptotic super-reflexive spaces, which turned out to be exactly the new isomorphic class of spaces with an equivalent norm with property (β). In a paper from 2012, Lima and Andrianarivony showed the importance of property (β) for solving a ten-year old question of Bates, Johnson, Lindenstrauss, Preiss and Schechtman about uniform (nonlinear) quotients. Independently, also in 2012, Revalski and Zhivkov defined the notion of compact uniform convexity in connection to the study of metric projections, and that turned out to be (isometrically) the same as the property (β) of Rolewicz. That led to the introduction of asymptotic midpoint uniform convexity (AMUC), and later it was proved that there are no uniform bi-Lipschitz embeddings of some types of countably branching graphs(for example, countably branching diamond and Laakso graphs) into Banach spaces with an equivalent AMUC norm. The lecture will be a short survey of these results, including several papers of the author with different sets of coauthors, both from the first and the second period of this research.

Convergence of remote projections onto convex set

Tuesday, 13.00-13.30

Eva Kopecká
University of Innsbruck

We try to find a point in the intersection of closed convex sets by iterating the nearest point projection onto them. This works, if the sets are symmetric and we always project on the most distant set. We will discuss to what extent this assumptions can be dropped.

Let $\{C_\lambda\}_{\lambda\in\Lambda}$ be a family of closed and convex sets in a Hilbert space H. Assume the sets C_λ are also symmetric and $\{x_n\}_{n\in\mathbb{N}}$ is a sequence of remote projections onto them. This means, $x_0 \in H$, and x_{n+1} is the projection of x_n onto the most distant set C_λ. Then the sequence $\{x_n\}$ converges to a point in the intersection $\bigcap_{\lambda\in\Lambda} C_\lambda$. We give examples explaining to what extent the symmetry condition on the sets C_λ can be dropped.
Joint work with P. Borodin.

On slicely countably determined sets in Banach spaces

Tuesday, 15.30-16.10

Johann Langemets
University of Tartu

In [1], A. Avilés, V. Kadets, M. Martín, J. Merí and V. Shepelska introduced the concept of slicely countably determined Banach spaces in order to generalize separable Banach spaces which are Asplund or have the Radon–Nikodým property. A bounded convex subset A of Banach space X is called *slicely countably determined* (an SCD set in short), if there exists a determining sequence of slices of A. Moreover, a separable Banach space X is called *slicely countably determined* (an SCD space in short), if every bounded convex subset of X is an SCD set.

We will first survey known results and some open problems around SCD sets and spaces. Then we will introduce and investigate a pointwise version of SCD sets, allowing us to extend this concept to non-separable spaces. Studying SCD points in detail enables us to prove some notable results in the context of separable spaces as well.

This is an ongoing joint work with M. Lõo, M. Martín, and A. Rueda Zoca.

References

[1] A. Avilés, V. Kadets, M. Martín, J. Merí, and V. Shepelska, *Slicely countably determined Banach spaces*, Trans. Am. Math. Soc., 362 (2010), pp. 4871–4900.

On the complemented subspace problem

Friday, 11.40-12.10

Gonzalo Martínez-Cervantes
Universidad de Alicante

The well-known Complemented Subspace Problem for $C(K)$-spaces, which asks whether every complemented subspace of a $C(K)$-space is also isomorphic to a $C(K)$-space, has been recently solved by G. Plebanek and A. Salguero-Alarcón in the negative.

The goal of this talk is to sketch the main ideas behind their construction and to show that their space not only fails to be isomorphic to a $C(K)$-space but it is not even isomorphic to a Banach lattice.

This talk is based on a work in progress with David de Hevia, Alberto Salguero-Alarcón and Pedro Tradacete.

A nonlinear Enflo example

Tuesday, 16.20-16.40

Rubén Medina
Universidad de Granada

In this talk we will follow the path traced by Per Enflo and give an explicit definition of a separable Banach space X without L-Lipschitz retractions onto any compact convex and generating subset of X, for some $L > 1$. This constitutes a first step towards the construction of a counterexample to Godefroy−Ozawa question, namely, a separable Banach space without Lipschitz retractions onto any compact convex and generating subset.

Separable Faces and Renormings of Non–separable Banach Spaces.
Nine Open Problems since 1975-76, 2007 and 2020.

Friday, 10.30-11.00

José Orihuela
Universidad de Murcia

We analyze question 18 of J. Lindenstrauss in [5]. We prove that a Banach space E with a norming subspace $F \subset E^*$ has an equivalent $\sigma(E, F)$-lower semicontinuous LUR norm if, and only if, there is a sequence $\{A_n : \ n = 1, 2, \cdots\}$ of subsets of E such that, given any $x \in E$ and $\epsilon > 0$, there is a $\sigma(E, F)$-open half-space H and $p \in \mathbb{N}$ such that $x \in H \cap A_p$ and the slice $H \cap A_p$ can be covered with countable many sets of diameter less than ϵ. Thus E has an equivalent $\sigma(E, F)$-lower semicontinuous LUR norm if, and only if, it has another one with separable denting faces, [9, 10] **This result completely solves four problems asked in [7, Question 6.33, p.128] extending Troyanski's fundamental results (see Chapter IV in [1]), and others ones in [3,6]. Moreover, LUR renormings are possible at points of separable faces wich could be glued as a σ-slicely isolated family of faces [7], of the unit sphere of E.** Among new examples covered by this results are Banach spaces $C(K)$, where K is a Rosenthal compact space $K \subset \mathbb{R}^\Gamma$ i.e., a compact space of Baire one functions on a Polish space Γ, with at most countably many discontinuity points for every $s \in K$, **which solves three problems asked in [7,Question 6.23, p.125].** Previously, it was only known for K being separable too, see [4] where the σ-fragmentability of $C(K)$ was already proved for non separable K, and a conjecture for the pointwise lower semicontinuous and LUR renorming presented here was posed, details will appear in [8].

For strictly convex renormings **we solve a recent question of R. Smith [11] giving a final answer to Lindenstauss question 18 in [5]**, see [2].

Joint work with V. Montesisnos. Research partially supported by Project PID2021-122126NB-C33 and Fundación Séneca - ACyT Región de Murcia project 21955/PI/22.

References

[1] R. Deville, G. Godefroy, and V. Zizler. *Smoothness and renormings in Banach spaces*, volume 64 of *Pitman Monographs and Surveys in Pure and Applied Mathematics.* Longman Scientific & Technical, Harlow, 1993.

[2] S. Ferrari, V. Montesinos and J. Orihuela. A separation method through convex sets for strictly convex renormings of Banach spaces. Preprint 2023.

[3] F. García, L. Oncina, J. Orihuela, and S. Troyanski. Kuratowski's index of non-compactness and renorming in Banach spaces. *J. Convex Anal.*, 11(2):477–494, 2004.

[4] R. Haydon, A. Moltó, and J. Orihuela. Spaces of functions with countably many discontinuities. *Israel J. Math.*, 158:19–39, 2007.

[5] J. Lindenstrauss. Some open problems in Banach space theory. In *Séminaire Choquet. Initiation à l' analyse tome 15, 1975-1976, Exp. No. 18*, pages 1–9. Secrétariat mathématique, Paris, 1975-76.
[6] A. Moltó, J. Orihuela, and S. Troyanski. Locally uniformly rotund renorming and fragmentability. *Proc. London Math. Soc. (3)*, 75(3):619–640, 1997.
[7] A. Moltó, J. Orihuela, S. Troyanski, and M. Valdivia. *A nonlinear transfer technique for renorming*, volume 1951 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 2009.
[8] A. Moltó, V. Montesinos, J. Orihuela and S. Troyanski. Rosenthal compact spaces and renormings. Work in progress.
[9] V. Montesinos and J. Orihuela. Separable slicing and locally uniformly rotund renormings of Banach spaces. Submitted January 2023
[10] V. Montesinos and J. Orihuela. Weak compactness and separability of faces for convex renormings of Banach spaces. Preprint 2023
[11] R.J. Smith A topological characterization of dual strict convexity in Asplund spaces. J. Math. Anal. Appl. 485 (2020) 123806

Dvoretzky-type theorem for locally finite subsets of a Hilbert space

Friday, 13.00-13.30

Mikhail Ostrovskii
St. John's University

A metric space is called *locally finite* if each ball of finite radius in it has finitely many elements.

The main result of the talk: If a locally finite metric space embeds isometrically into a Hilbert space, then it embeds almost isometrically into an arbitrary infinite-dimensional Banach space.

I shall present some background on this topic and some ideas of the proof. I intend to make my talk accessible for non-experts.

Based on a joint with F. Catrina and S. Ostrovska paper.

On the weak topology in Lipschitz free spaces

Wednesday, 12.50-13.30

Colin Petitjean
Université Gustave Eiffel

For a metric space (M,d), the Lipschitz free space (also known as Arens–Eells space or transportation cost space) $\mathcal{F}(M)$ is a Banach space which is built around M in such a way that M is isometric to a (linearly dense) subset $\delta(M)$ of $\mathcal{F}(M)$, and Lipschitz maps from $\delta(M)$ into any Banach space X uniquely extend to bounded linear operators from $\mathcal{F}(M)$ into X. The study of free spaces is at the intersection of functional analysis, metric geometry, measure theory, transportation theory, etc. A recent program consists in trying to characterise (linear) properties of $\mathcal{F}(M)$ in terms of (metric) properties of M. In this talk, we will present a few results concerning the weak topology in Lipschitz free spaces.

On the numerical index of the real two-dimensional L_p space

Tuesday, 16.45-17.05

Alicia Quero de la Rosa
Universidad de Granada

The concept of numerical index was introduced by Lumer in 1968 in the context of the study and classification of operator algebras. This is a constant relating the norm and the numerical range of bounded linear operators on the space. Namely, the numerical index of a Banach space X, $n(X)$, is the greatest constant $k \geq 0$ such that

$$k\|T\| \leq \sup\bigl\{|x^*(Tx)| \colon x \in X,\ x^* \in X^*,\ \|x\| = \|x^*\| = x^*(x) = 1\bigr\}$$

for every operator $T \in \mathcal{L}(X)$.

There are some classical Banach spaces for which the numerical index has been calculated, however the problem of computing the numerical index of L_p-spaces when $p \neq 1, 2, \infty$ has been latent since the beginning of the theory. The aim of this talk is to give an overview of the state of this problem, paying special attention to the two-dimensional real case for which we have recently been able to give a partial solution. More precisely, we have calculated the numerical index of the two-dimensional real L_p-space for $6/5 \leq p \leq 6$. ([1],[2]).

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Different convexity concepts in norms of Banach spaces

Wednesday, 17.00-17.20

Andrés Quilis
Universitat Politècnica de València / Czech Technical University in Prague

We study several classical concepts in the topic of strict convexity of norms in infinite dimensional Banach spaces. Specifically, and in descending order of strength, we deal with Uniform Rotundity (UR), Weak Uniform Rotundity (WUR) and Uniform Rotundity in Every Direction (URED). We describe the geometrical intuition behind these concepts, and show that we may distinguish between all of these three properties in every Banach space where such renormings are possible.

Distances between $C(K)$ spaces

Thursday, 16.10-16.30

Jakub Rondoš
Charles University Prague

It has been known for some time that distances between Banach spaces of continuous functions on compact Hausdorff spaces have some very unexpected behavior. For example, the classical result of Amir and Cambern states that there are no compact spaces K, L such that the Banach-Mazur distance between $C(K)$ and $C(L)$ is strictly between 1 and 2. This has been quite recently generalized in some sense for nonlinear distances. Further, an old conjecture of Pelczynski, which states that the Banach-Mazur distance between two isomorphic $C(K)$ spaces is always an integer, still remains open. In the talk, we survey the known results about various types of distances between $C(K)$ spaces.

Norming tales

Monday, 15.30-16.10

Tommaso Russo
Universität Innsbruck

Markushevich bases (M-bases, for short) are an extremely handy tool in Banach space theory. Compared to the more common notion of Schauder bases, they offer two clear advantages: firstly, they exist in every separable Banach space and second their definition is equally well suited for non-separable Banach spaces. For there reasons their existence and properties have been intensely studied and several classes of Banach spaces have been detected, that can be characterised by the existence of M-bases with specific properties.

In the talk we shall focus on norming M-bases, namely M-bases where the linear span of the coordinate functionals is a norming subspace. We will explain their connection with WCG Banach spaces and survey some recent results in the area. Along the way, we will also mention some open problems and possible directions for further research.

Continuous operators from spaces of Lipschitz functions

Thursday, 16.35-16.55

Damian Sobota
Kurt Gödel Research Center, Vienna University

For a metric space M by $\mathrm{Lip}_0(M)$ we denote the Banach space of Lipschitz real-valued functions on M vanishing at a fixed point 0, endowed with the Lipschitz constant norm. During my talk we will discuss the existence of continuous linear surjections between spaces of the form $\mathrm{Lip}_0(M)$ and $(\mathrm{Lip}_0(N), \tau)$, where τ denotes the weak topology or the pointwise topology of $\mathrm{Lip}_0(N)$. We will also be interested in the question when a given space $\mathrm{Lip}_0(M)$ admits bounded linear operators onto the classical Banach spaces c_0 and ℓ_1, and compare the situation to the case of the Banach spaces $C(K)$ of continuous real-valued functions on compact spaces K.

(Joint work with C. Bargetz and J. Kakol.)

Some recent progresses in renorming theory

Wednesday, 16.10-16.30

Jacopo Somaglia
Politecnico di Milano

In a recent monograph A.J. Guirao, V. Montesinos, and V. Zizler posed several interesting problems on renorming techniques in Banach spaces. The present talk is dedicated to those problems which have been solved recently, with particular attention to the following one: *does every infinite-dimensional separable Banach space admit a norm that is rotund and Gâteaux smooth but not locally uniformly rotund?*

It turns out that the above question admits a positive answer, as we showed in collaboration with C.A. De Bernardi.

Unconditional bases and Daugavet renormings

Wednesday, 16.35-16.55

Triinu Veeorg
University of Tartu

In this talk, we consider renormings of Banach spaces that produce Daugavet points. In particular we show that every infinite dimensional Banach space with an unconditional Schauder basis can be renormed to have a Daugavet point.

Posters

The lattice Lipschitz inequality on $\mathcal{C}(K)$

Andres Roger Arnau Notari
Universidad Politécnica de Valencia

Given a compact topological space K, we present the class of lattice Lipschitz (non-linear) operators from a subset of $\mathcal{C}(K)$ to $\mathcal{C}(K)$. These operators satisfy a lattice inequality on which the Lipschitz constants becomes a continuous function on K. They provide a natural Lipschitz generalization of the linear notions of diagonal operator and multiplication operators on Banach function lattices. We establish a McShane extension type theorem, that allow to extend those operators from a set of continuous functions to the entire $\mathcal{C}(K)$ space.

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Small Diameter Properties in Banach spaces

Sudeshna Basu
Loyola University

The geometry of Banach space is an area of research which charecterizes the topological and measure theoretic concepts in Banach spaces in terms of geometric structure of the space. In this work we study three different versions of small diameter properties of the unit ball in a Banach space and its dual. The related concepts for all closed bounded convex sets of a Banach space was initiated developed and extensively studied in the context of Radon Nikodym Property and Krein Milman Property in [1] and developed subsequently. We prove that all these properties are stable under l_p sum for $1 \leq p \leq \infty$, c_0 sum and Lebesgue Bochner spaces. We show that these are three space properties under certain conditions on the quotient space. We also study these properties in ideals of Banach spaces. This is based on two papers jointly written with my graduate student, Susmita Seal [2], [3].

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Zero product determined Banach algebras and Jordan homomorphisms

María Luisa Castillo Godoy
Universidad de Granada

A (complex) Banach algebra A is said to be *zero product determined* if every continuos bilinear map $\varphi\colon A \to \mathbb{C}$ with the property that $\varphi(a,b) = 0$ whenever $ab = 0$ is of the form $\varphi(a,b) = \tau(ab)$ for some continuous linear map $\tau \in A^*$.

A rather wide class of Banach algebras that includes C^*-algebras and group algebras, among others, are zero product determined, and we use this property to characterize Jordan homomorphisms on these algebras. More precisely, we show that an operator T from a Banach algebra A onto a Banach algebra with BAI is a Jordan homomorphism in any of the following cases:

- A is a C^*-algebra or the algebra of approximable operators on a Banach space which dual has the bounded approximation property and T satisfies $T(a)T(b) + T(b)T(a) = 0$ if $ab = ba = 0$.
- A is a group algebra associated with a compact group and T satisfies $T(a)T(b) = T(b)T(a) = 0$ if $ab = ba = 0$.

Sobolev maps with values in an arbitrary metric space

Nikita Evseev
Harbin Institute of Technology

We are concerned here with Sobolev-type space of functions valued in Banach space or in metric space. We review two ways of defining Sobolev map valued in metric space: Reshetnyak's approach vs definition via postcomposition with the Kuratowskii embedding. In particular we show that Sobolev map with values in dual Banach space can be described in terms of classical weak derivatives in weak* sense.

Planes in Schatten-p, and a problem of Ball, Carlen and Lieb

Otte Heinaevaara
Princeton University

In his seminal 1956 paper Olof Hanner proved an inequality, now known as Hanner's inequality, that captures the modulus of uniform convexity of L_p. Similar inequality was later considered by Ball, Carlen and Lieb in 1994 for non-commutative variants of L_p, Schatten-p, and while they managed to sidestep this defect, Ball, Carlen and Lieb only managed to prove Hanner's inequality for Schatten-p when $p \geq 4$ or $1 \leq p \leq 4/3$.

We propose a new conjecture on Schatten-p spaces: every two-dimensional subspace of Schatten-p is linearly isometric to a subspace of L_p. This would immediately imply Hanner's inequality for Schatten-p, together with wide range of other inequalities. We prove this conjecture when $p = 3$ using some curious matrix identities.

On the differentiability of the convolution

Pablo Jiménez Rodríguez
Universidad de Valladolid

There are several classic results that convince us that the convolution operator is a smoothening one, in the sense that the properties of the resulting function are better than those of the parent functions. In this poster we present two differentiable functions whose convolution fails to be differentiable in certain points.

Topological entropy for multivalued mappings

Pavel Ludvík
Palacký University Olomouc

The topological entropy, defined for single-valued continuous maps by Adler, Konheim and McAndrew in 1965 in compact topological spaces, and by Bowen in 1971 in metric spaces, measures a complexity of behaviour to related dynamical systems. Its positive value is sometimes characterized as *topological chaos*.

Recently, the study of topological entropy was also extended to multivalued maps in metric spaces (by authors Kelly, Tennant; Carrasco-Olivera, Metzger Alvan, Morales Rojas; Andres, Ludvík; ...).

In my poster I would like to provide a brief summary of evolution of the concept of topological entropy and introduce several results of my joint work with Jan Andres. We are concerned with estimates of multivalued topological entropy; entropy of induced hypermaps; generalizations to uniform spaces, etc.

Lipschitz-free p-space norm

Tomáš Raunig
Charles University

Let $0 < p \leq 1$, M be a p-metric space and $\mathcal{F}_p(M)$ be its Lipschitz-free p-space (for $p = 1$ this is exactly the standard Lipschitz-free space, for $0 < p < 1$ it is an analogue in the setting of p-metric and p-Banach spaces). We present a new way to express the Lipschitz-free p-norm which, in the case that M is finite, gives a finite algorithm for calculating the p-norm of any element. As consequences, first we give a theorem which demonstrates the fundamental difference between the case $p = 1$ and $p < 1$. Second, we show how this result can be applied to the problem of p-amenability of (p-)metric spaces.

The poster is based on a work in progress with Marek Cúth.

Haar null convex sets

Davide Ravasini
University of Innsbruck

A Borel set E in an abelian Polish group X is Haar null if there is a Borel probability measure μ on X such that $\mu(x+E)=0$ for every $x \in X$. Haar null sets are a generalisation of sets with zero Haar measure to nonlocally compact, abelian Polish groups and, as in the locally compact case, form a translation-invariant σ-ideal. We present several geometric, measure-free characterisations of Haar null closed, convex sets in separable Banach spaces, generalising the well-known fact that a closed, convex subsets of $\mathbb{R}^d$ has zero Lebesgue measure if and only if it has empty interior.

Mazur-Ulam property of JB*-algebras

Radovan Švarc
Charles University

JB*-algebras are generalizations of C*-algebra, exchanging the not necessarily commutative setting of associative algebras for commutative but not necessarily associative setting of Jordan algebras. In the paper behind this poster, we present a proof that unital JB*-algebras have Mazur-Ulam property, i.e. whenever X is a unital JB*-algebra, Y is a Banach space, and T is an isometry of unit sphere $S(X)$ of X onto unit sphere $S(Y)$ of Y, then T can be extended to a linear isometry mapping X onto Y. To prove this we had to discover some new properties of JB*-algebras. Probably the most interesting one of them says that if X is a unital JB*-algebra and e is a minimal tripotent in the bidual $X**$, then there is a self-adjoint element h in X such that $e \leq \exp(ih)$.

This is a joint work with Antonio M. Peralta (Universidad de Granada). The presenting author is supported by the Charles University, project GAUK no. 268521.
